

Lorentz invariant dark-spinor and inflation

Abhishek Basak[†] and Jitesh R. Bhatt[‡]

*Theoretical Physics Division, Physical Research Laboratory,
Navarangpura, Ahmedabad, India*

E-mail: abhishek@prl.res.in, jeet@prl.res.in

ABSTRACT: We investigate the possibility of the inflation driven by a Lorentz invariant non-standard spinor field. As these spinors are having dominant interaction via gravitational field only, they are considered as *Dark Spinors*. We study how these dark-spinors can drive the inflation and investigate the cosmological (scalar) perturbations generated by them. Though the dark-spinors obey a Klein-Gordon like equation, the underlying theory of the cosmological perturbations is far more complex than the theories which are using a canonical scalar field. For example the sound speed of the perturbations is not a constant but varies with time. We find that in order to explain the observed value of the spectral-index n_s one must have upper bound on the values of the background NSS-field. The tensor to scalar ratio remains as small as that in the case of canonical scalar field driven inflation because the correction to tensor spectrum due to NSS is required to be very small. In addition we discuss the relationship of results with previous results obtained by using the Lorentz invariance violating theories.

KEYWORDS: dark spinors, non-standard spinors, inflation, cosmological-perturbations, Lorentz invariance

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1 Introduction

High precision cosmology has significantly changed our idea of the universe. The present state of the universe can be explained very well with the assumption that very early during its history the universe has undergone an inflationary phase of the expansion [1–4]. In addition the measurements of Type-I supernovae around redshift $z \sim 1$, together with the other measurements, suggest that the universe is currently undergoing an accelerated phase of expansion for the second time in its history after the big bang. This phase of the accelerated expansion is attributed to dark-energy [5, 7]. According to our present understanding the total energy of the universe is very close to the critical energy and the baryonic matter constitute only 4% of it. While the dark-matter and dark-energy contribute around 22% and 74% of the total energy respectively [6]. Presence of the dark-sector in the energy budget may be indicative of the fact that either general relativity or the standard model of particles (or both) are inadequate to explain the current astrophysical and cosmological data. Thus in such a situation it is worthwhile to look for new kind of particles or fields that can be candidates for the dark-matter, dark-energy and inflation.

Recently there is a lot of interests in studying dark or Non-Standard Spinor (NSS). The theory of NSS was first developed in Refs.[8, 9]. Subsequently the NSS models were further developed and investigated by several authors [10–16]. These spinors can be regarded as ‘dark’ as their dominant interaction is via gravitational field only and they have been extensively applied to study above mentioned problems in cosmology [17–27]. Unlike the Dirac, Majorana or Weyl spinors, NSS propagator behaves like $1/p^2$ in the large momentum limit and has mass dimension one. At present the theory of NSS is under development, however, NSS are known to be either violating the Lorentz invariance or locality or both. Basic Lagrangian of NSS can be written as

$$\mathcal{L}_{\text{cosmo}} = \frac{1}{2} \vec{\lambda} \overleftarrow{\nabla}_\mu \nabla^\mu \lambda - V(\vec{\lambda} \lambda), \quad (1.1)$$

where, $\bar{\lambda} \overleftarrow{\nabla}_\mu \equiv \partial_\mu \bar{\lambda} + \bar{\lambda} \Gamma_\mu$, $\nabla_\mu \lambda \equiv \partial_\mu \lambda - \Gamma_\mu \lambda$. λ and $\bar{\lambda}$ are NSS and its dual respectively. Γ_μ are defined as

$$\Gamma_\mu = \frac{i}{4} \omega_\mu^{ab} f_{ab}, \quad f^{ab} = \frac{i}{4} [\gamma^a, \gamma^b], \quad (1.2)$$

where index μ is the space-time index and index a is the spinor index. Here ω_μ^{ab} is defined as

$$\omega_\mu^{ab} = e_\nu^a \partial_\mu e^\nu b + e_\nu^a e^\sigma b \Gamma_{\mu\sigma}^\nu,$$

where e_μ^a are tetrads defined as $e_\mu^a e_\nu^b \eta_{ab} = g_{\mu\nu}$. Here $g_{\mu\nu} = a^2 \text{diag}(1, -1, -1, -1)$ is the space-time metric, a is the scale factor, $\eta_{ab} = \text{diag}(1, -1, -1, -1)$ and $\Gamma_{\mu\sigma}^\nu$ are Christoffel symbols of $g_{\mu\nu}$. γ -matrices are defined as

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{I}_{2 \times 2} \\ \mathbb{I}_{2 \times 2} & 0 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & -\sigma^i \\ \sigma^i & 0 \end{pmatrix},$$

where σ^i ($i = 1, 2, 3$) are Pauli matrices defined as

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

It should be noted that the NSS theory described by the above Lagrangian is not Lorentz invariant [17]. It has been proposed in [17] that the locally Lorentz invariant energy-momentum tensor $T_{\text{cosmo}}^{\mu\nu}$ can be constructed from $\mathcal{L}_{\text{cosmo}}$ as

$$T_{\text{cosmo}}^{\mu\nu} = \bar{\lambda} \overleftarrow{\nabla}^{(\mu} \nabla^{\nu)} \lambda - g^{\mu\nu} \mathcal{L}_{\text{cosmo}} + \frac{1}{2} \nabla_\rho J^{\mu\nu\rho}, \quad (1.3)$$

where $J^{\mu\nu\rho}$ defined as

$$J^{\mu\nu\rho} = -i \left[\bar{\lambda} \overleftarrow{\nabla}^{(\mu} f^{\nu)\rho} \lambda + \bar{\lambda} f^{\rho(\mu} \nabla^{\nu)} \lambda \right] \quad (1.4)$$

is the additional term which did not appear in the earlier non-Lorentz invariant models [21–24]. It can be argued that any reasonable theory of cosmological perturbations should be based on Lorentz invariant frame work. Keeping this view in mind we study the characteristics of the inflation driven by NSS given in [17]. As it turned out, the cosmological perturbations based on equation(1.3) are far more complex than the theory based upon canonical scalar field model. Appearance of $J^{\mu\nu\rho}$ term can give rise to an additional scale $\tilde{F} = \frac{\bar{\lambda} \lambda}{8M_{\text{pl}}^2}$ in the problem, where $M_{\text{pl}} = \sqrt{\frac{1}{8\pi G}}$ is the reduced Planck mass (G is the gravitational constant).

It is generally assumed that the inflation is driven by a scalar field, which can have the following verifiable predictions: (a) nearly a scale invariant spectrum, (b) existence of gravitational waves and (c) the tensor to scalar power spectrum may be of the order of ϵ , where ϵ is the slow roll parameter [2–4]. At this juncture in order to gain further insight it is natural to question the elementary scalar field inflation scenario. Therefore, in this work, we investigate some of the predictions of the inflation theory by assuming that the inflation is driven by a NSS with energy-momentum tensor described by equation (1.3).

We would like to note that a similar kind of study exist in the literatures but it is based on non-Lorentz invariant NSS [18].

In this study we are interested in calculating power spectrum of the scalar perturbations generated by NSS and compare our results with the spectrum calculated from canonical scalar inflaton field. In section 2 we write the basic equations for homogeneous and isotropic background. In section 3 we give the details of the cosmological perturbations using $T^{\mu\nu}$ given in equation (1.3). While section 4 contains the calculation of power spectrum and its comparison with CMB spectrum.

2 Unperturbed equations

Here we consider the following structure of NSS (λ) and its dual $(\bar{\lambda})$:

$$\lambda = \varphi(\eta)\xi, \quad \bar{\lambda} = \varphi(\eta)\bar{\xi}, \quad (2.1)$$

where, $\varphi(\eta)$ is a scalar quantity and η is the conformal time defined as $d\eta = \frac{dt}{a}$. ξ and $\bar{\xi}$ are two constant matrices with $\bar{\xi}\xi = \mathbb{I}$. In a flat, isotropic and homogeneous space-time unperturbed $\mathcal{L}_{\text{cosmo}}$ can be written as

$$\mathcal{L}_{\text{cosmo}} = \frac{1}{2a^2} \left[\varphi'^2 + \frac{3}{4} H^2 \varphi^2 \right] - V(\varphi^2), \quad (2.2)$$

where prime $(')$ denotes the derivative with respect to conformal time η . $V(\varphi)$ is the potential which is a function of φ . While the Hubble expansion parameter H is defined as $H = \frac{a'}{a}$. Unperturbed energy momentum tensors and equation of motion for φ in FRLW space-time have already been calculated in [17]. We are enlisting them below in conformal time. Let us first define the covariant energy momentum tensor $T_{\text{cosmo}}^{\mu\nu}$, which appears into the Einstein's equation, as

$$T_{\text{cosmo}}^{\mu\nu} = \bar{T}^{\mu\nu} + \frac{1}{2} \nabla_\rho J^{\mu\nu\rho}, \quad (2.3)$$

where,

$$\bar{T}^{\mu\nu} = \bar{\lambda} \overleftarrow{\nabla}^{(\mu} \nabla^{\nu)} \lambda - g^{\mu\nu} \mathcal{L}_{\text{cosmo}}.$$

The non-vanishing components of $J^{\mu\nu\rho}$ are

$$J^{i\eta j} = J^{\eta ij} = \frac{1}{4} \frac{H}{a^4} \varphi^2 \delta_{ij}, \quad J^{ij\eta} = -\frac{1}{2} \frac{H}{a^4} \varphi^2 \delta_{ij}.$$

Here $\varphi^2 = \bar{\lambda}\lambda$, is a function of time only. Next, we can write the expressions for the energy density ε and pressure p as following:

$$\varepsilon = T_\eta^\eta = \bar{T}_\eta^\eta + F_\eta^\eta, \quad p = -T_j^i \delta_{ij} = -\left(\bar{T}_j^i + F_j^i\right) \delta_{ij}.$$

Expressions for $\bar{T}^{\mu\nu}$ and $F^{\mu\nu}$ can be written as,

$$\bar{T}_\eta^\eta = \frac{1}{2a^2} \left[\varphi'^2 - \frac{3}{4} H^2 \varphi^2 \right] + V$$

and

$$F_\eta^\eta = \frac{3}{4a^2} H^2 \varphi^2.$$

From these one can write energy density as

$$\varepsilon = \frac{1}{2a^2} \left[\varphi'^2 + \frac{3}{4} H^2 \varphi^2 \right] + V. \quad (2.4)$$

It is useful to write the expression for ε as,

$$\varepsilon = X + V,$$

where, $X = \left(\nabla^\eta \bar{\lambda} \nabla_\eta \lambda \right) - g_{\eta\eta} \left(\frac{1}{2} \nabla_\mu \bar{\lambda} \nabla^\mu \lambda \right) + g_{\eta\eta} \frac{1}{2} \nabla_\rho J^{\eta\eta\rho} = \frac{1}{2a^2} \left[\varphi'^2 + \frac{3}{4} H^2 \varphi^2 \right]$. Considering the diagonal space-space components of energy-momentum tensor one can write

$$\bar{T}_j^i \delta_{ij} = -\frac{1}{2a^2} \left[\varphi'^2 + \frac{1}{4} H^2 \varphi^2 \right] + V,$$

and

$$F_j^i \delta_{ij} = \frac{1}{4a^2} H^2 \varphi^2 + \frac{1}{4a^2} (H\varphi^2)'.$$

From these one can obtain the expression for pressure as

$$p = \frac{1}{2a^2} \left[\varphi'^2 - \frac{1}{4} H^2 \varphi^2 \right] - \frac{1}{4a^2} (H\varphi^2)' - V. \quad (2.5)$$

It is easy to notice that the pressure is homogeneous and isotropic. All other components of background T_ν^μ are zero. By adding ε and p

$$(\varepsilon + p) = \frac{\varphi'^2}{a^2} + \frac{1}{4a^2} H^2 \varphi^2 - \frac{1}{4a^2} (H\varphi^2)'. \quad (2.6)$$

For the instance when the last two terms in the above equations are absent, one can recover the expression for $(\varepsilon + p)$ of the canonical scalar-field. Equation of motion for φ can be obtained by equating the divergence of T_ν^μ to zero:

$$\varphi'' + 2H\varphi' - \frac{3}{4} H^2 \varphi + V_{,\varphi} = 0. \quad (2.7)$$

It should be emphasized that the above equation for φ matches with the equation motion obtained using Euler-Lagrange equation as discussed in [17]. However, in the earlier calculations based on non Lorentz invariant model of NSS there were mismatches between the equation motions calculated using these two methods, e.g. [23]. This is solved because of the additional term F_ν^μ in equation (1.3). The modified Friedmann equations can be written as

$$\begin{aligned} H^2 &= \frac{1}{1 - \tilde{F}} \left[\frac{1}{3M_{\text{pl}}^2} \left(\frac{\varphi'^2}{2} + a^2 V \right) \right], \\ H' &= \frac{1}{1 - \tilde{F}} \left[\frac{1}{3M_{\text{pl}}^2} (a^2 V - \varphi'^2) + H \tilde{F}' \right], \end{aligned} \quad (2.8)$$

where, $\tilde{F} = \frac{\varphi^2}{8M_{pl}^2}$. One can notice from the above that the condition $\tilde{F} < 1$ is required to be satisfied to ensure the positivity of H^2 . Therefore φ has to be smaller than $\sqrt{8}M_{pl}$ as mentioned in [17]. We would like to emphasize that the introduction of $J^{\mu\nu\rho}$ term in equation (1.3) makes the expressions for H^2 and H' different from those obtained in [18]. From what follows we drop the label *cosmo* on the energy-momentum tensor defined by equation (1.3).

3 Perturbed equations

In this work we closely follow the gauge invariant approach for treating the cosmological perturbations discussed in Ref. [3]. Total FRLW metric (perturbed + unperturbed) can be written as:

$$\bar{g}_{\mu\nu} + \delta g_{\mu\nu} = a^2 \begin{pmatrix} (1+2\psi) & \mathbb{O} \\ \mathbb{O} & \{(-1+2\phi)\delta_{ij} + 2h_{ij}\} \end{pmatrix}. \quad (3.1)$$

Here i, j denotes space-space components of the metric, ϕ, ψ are scalar perturbations and h_{ij} are traceless and divergence-less tensor perturbations. The metric perturbations are functions of space and time. We first calculate the perturbations in energy momentum tensor δT_ν^μ by including the perturbations in F_ν^μ term. Final equations are obtained by substituting δT_ν^μ into the perturbed Einstein's equations. Following the structure of unperturbed λ and $\bar{\lambda}$ in reference [18], we consider the following form of perturbed $\delta\lambda$ and its dual $\delta\bar{\lambda}$:

$$\delta\lambda = \frac{\varphi(\eta)}{\sqrt[4]{12}} \begin{pmatrix} -\alpha_1 e^{i\frac{\pi}{4}} \delta\varphi_1 \\ \alpha_2 \frac{i}{\sqrt{2}} \delta\varphi_2 \\ \alpha_2 \frac{1}{\sqrt{2}} \delta\varphi_3 \\ \alpha_1 e^{i\frac{\pi}{4}} \delta\varphi_4 \end{pmatrix} = \frac{\delta\varphi}{\sqrt[4]{12}} \begin{pmatrix} -\alpha_1 e^{i\frac{\pi}{4}} \\ \alpha_2 \frac{i}{\sqrt{2}} \\ \alpha_2 \frac{1}{\sqrt{2}} \\ \alpha_1 e^{i\frac{\pi}{4}} \end{pmatrix}$$

and

$$\begin{aligned} \delta\bar{\lambda} &= \frac{\varphi(\eta)}{\sqrt[4]{12}} \begin{pmatrix} -\alpha_1 e^{-i\frac{\pi}{4}} \delta\varphi_1 & -\alpha_2 \frac{i}{\sqrt{2}} \delta\varphi_2 & \alpha_2 \frac{1}{\sqrt{2}} \delta\varphi_3 & \alpha_1 e^{-i\frac{\pi}{4}} \delta\varphi_4 \end{pmatrix} \\ &= \frac{\delta\varphi}{\sqrt[4]{12}} \begin{pmatrix} -\alpha_1 e^{-i\frac{\pi}{4}} & -\alpha_2 \frac{i}{\sqrt{2}} & \alpha_2 \frac{1}{\sqrt{2}} & \alpha_1 e^{-i\frac{\pi}{4}} \end{pmatrix}, \end{aligned} \quad (3.2)$$

with $\delta\varphi_1 = \delta\varphi_2 = \delta\varphi_3 = \delta\varphi_4$, $\delta\varphi = \varphi\delta\varphi_1$. Here $\alpha_1 = \alpha_2^{-1} = \sqrt{\frac{1+\sqrt{3}}{2}}$. $\delta\varphi$ is a function of space and time. It should be noted that here we have not used the hedgehog ansatz for the unperturbed NSS like the previous study [18]. Instead with the relatively simpler ansatz considered above one can check that all the equations of the cosmological perturbations given in [18] can be reproduced if the $J^{\mu\nu\rho}$ term is ignored from equation (1.3).

3.1 Perturbed energy momentum tensors:

Using equations (3.1, 3.2) we can calculate δT_η^η , δT_i^η and $\delta T_j^i (i \neq j)$ components of the perturbed energy-momentum tensor. Below we have enlisted the different components of the energy-momentum tensor for the scalar perturbations.

i) Perturbation of $\varepsilon = T_\eta^\eta$: One can write the general expression for energy as

$$\varepsilon = X + V,$$

where, X can be written as $X = Y + g_{\eta\eta} \frac{1}{2} \nabla_\rho J^{\eta\eta\rho}$, here, $Y = (\nabla^\eta \bar{\lambda} \nabla_\eta \lambda) - g_\eta^\eta \left(\frac{1}{2} \nabla_\mu \bar{\lambda} \nabla^\mu \lambda \right)$. From the expression of ε which is a function of X and V we can write

$$\delta\varepsilon = \varepsilon_{,X} \delta X + \varepsilon_{,\varphi} \delta\varphi$$

and from continuity equation we know

$$\varepsilon' = \varepsilon_{,X} X' + \varepsilon_{,\varphi} \varphi' = -3H(\varepsilon + p).$$

Eliminating $\varepsilon_{,\varphi}$ from those two equations we get

$$\delta\varepsilon = \varepsilon_{,X} \left(\delta X - X' \frac{\delta\varphi}{\varphi'} \right) - 3H(\varepsilon + p) \frac{\delta\varphi}{\varphi'}.$$

The perturbation in Y is

$$\delta Y = \frac{1}{a^2} \left(-\psi \varphi'^2 + \frac{3}{4} \psi H^2 \varphi^2 + \varphi' \delta\varphi' + \frac{3}{4} \psi' H \varphi^2 - \frac{3}{4} H^2 \varphi \delta\varphi \right),$$

while the perturbation in $F^{\eta\eta}$ can be written as

$$\delta F^{\eta\eta} = \frac{1}{2} \delta (\nabla_\rho J^{\eta\eta\rho}).$$

Next, the perturbation in the covariant derivative of $J^{\mu\nu\rho}$ can be written as;

$$\delta (\nabla_\rho J^{\eta\eta\rho}) = \partial_\rho \delta J^{\eta\eta\rho} + \delta (\Gamma_{\sigma\rho}^\eta J^{\sigma\eta\rho} + \Gamma_{\sigma\rho}^\eta J^{\eta\sigma\rho} + \Gamma_{\sigma\rho}^\rho J^{\eta\eta\sigma}).$$

Therefore we get after substituting for $\delta (\nabla_\rho J^{\eta\eta\rho})$

$$\delta F^{\eta\eta} = -\frac{1}{4a^4} (\Delta\psi) \varphi^2 + \frac{3}{2a^4} H^2 \varphi \delta\varphi - \frac{3}{2a^4} \phi' H \varphi^2 - \frac{3}{a^4} \psi H^2 \varphi^2.$$

From this one can calculate δX

$$\begin{aligned} \delta X &= -\psi (2X) + \frac{1}{a^2} \varphi' \delta\varphi' + \frac{3}{4a^2} (\psi' - 2\phi') H \varphi^2 + \frac{3}{4a^2} H^2 \varphi \delta\varphi - \frac{1}{4a^2} (\Delta\psi) \varphi^2, \\ X' &= -2HX + \frac{1}{a^2} \varphi' \varphi'' + \frac{3}{4a^2} HH' \varphi^2 + \frac{3}{4a^2} H^2 \varphi \varphi'. \end{aligned}$$

Finally one can write the energy perturbation $\delta\varepsilon$ as

$$\begin{aligned} \delta\varepsilon &= \varepsilon_{,X} [2X \left(-\psi + H \frac{\delta\varphi}{\varphi'} + \left(\frac{\delta\varphi}{\varphi'} \right)' \right) - \frac{3}{4a^2} H \varphi^2 \left(H \frac{\delta\varphi}{\varphi'} \right)' + \frac{3}{4a^2} (\psi' - 2\phi') H \varphi^2 - \\ &\quad \frac{1}{4a^2} (\Delta\psi) \varphi^2] - 3H(\varepsilon + p) \frac{\delta\varphi}{\varphi'}. \end{aligned} \tag{3.3}$$

ii) Perturbation of T_i^η :

$$\delta T_i^\eta = \delta \bar{T}_i^\eta + \delta F_i^\eta.$$

Now for scalar perturbation

$$\delta\bar{T}_i^\eta = \left[\frac{1}{a^2}\varphi'\delta\varphi - \frac{1}{4a^2}(H\varphi^2)\psi \right]_{,i},$$

and

$$F_i^\eta = \left[-\frac{a^2}{8} \left(\frac{\psi\varphi^2}{a^4} \right)' - \frac{1}{8a^2}H(2\varphi\delta\varphi) - \frac{1}{4a^2}(\psi + \phi)H\varphi^2 + \frac{1}{8a^2}\phi'\varphi^2 \right]_{,i}.$$

And we get

$$\begin{aligned} \delta T_i^\eta = & \left[\frac{1}{a^2}\varphi'\delta\varphi - \frac{1}{4a^2}(H\varphi^2)\psi \right]_{,i} + \\ & \left[-\frac{a^2}{8} \left(\frac{\psi\varphi^2}{a^4} \right)' - \frac{1}{8a^2}H(2\varphi\delta\varphi) - \frac{1}{4a^2}(\psi + \phi)H\varphi^2 + \frac{1}{8a^2}\phi'\varphi^2 \right]_{,i}. \end{aligned} \quad (3.4)$$

iii) Perturbation of T_j^i ($i \neq j$):

$$\delta T_j^i = \delta\bar{T}_j^i + F_j^i.$$

Now for scalar perturbation, $\delta\bar{T}_j^i = 0$ and $F_j^i = -\frac{1}{4a^2}(\partial_i\partial_j\phi)\varphi^2$ for $i \neq j$. Therefore

$$\delta T_j^i = -\frac{1}{4a^2}(\partial_i\partial_j\phi)\varphi^2 \quad (i \neq j). \quad (3.5)$$

3.2 Perturbed Einstein's Equation:

Perturbed Einstein's equation can be written as:

$$\delta G_\nu^\mu = 8\pi G\delta T_\nu^\mu,$$

where δG_ν^μ is the perturbed Einstein's tensor. The scalar part of perturbed Einstein's equations are given below,

$$\Delta\phi - 3H(\phi' + H\psi) = 4\pi G a^2 \delta T_\eta^\eta$$

$$\begin{aligned} & - \left[2\phi'' + 2\frac{a'}{a}(\psi' + 2\phi') - 2 \left\{ \left(\frac{a'}{a} \right)^2 - 2\frac{a''}{a} \right\} \psi + \Delta(\psi - \phi) \right] \delta_{ij} + \partial_i \partial_j (\psi - \phi) = 8\pi G a^2 \delta T_j^i \\ & (\phi' + H\psi)_{,i} = 4\pi G a^2 \delta T_i^\eta, \end{aligned} \quad (3.6)$$

In the previous sub-section we have already calculated the scalar perturbations for the various components of the energy-momentum tensor. The tensor part of the perturbed Einstein's equations can be written as,

$$h_{ij}'' + 2Hh_{ij}' - \Delta h_{ij} = -16\pi G a^2 \delta T_{j(T)}^i,$$

where subscript T on the energy-momentum tensor denotes the tensor part. Next, consider the space-space components of the Einstein equation with $(i \neq j)$.

i) $\delta G_j^i = 8\pi G \delta T_j^i$: Using the expression of δT_j^i when $i \neq j$ from equation (3.5) one can write,

$$\partial_i \partial_j (\psi - \phi) = \partial_i \partial_j (-2\tilde{F}\phi),$$

where $\tilde{F} = \pi G \varphi^2 = \frac{\varphi^2}{8M_{\text{PL}}^2}$. In the case of the standard inflation driven by a canonical scalar-field, $\delta T_j^i = 0$ for $i \neq j$ and $\phi = \psi$. However, this is no longer true for a NSS driven inflation. The above equation implies that the condition $\psi = (1 - 2\tilde{F})\phi$ needs to be satisfied by the metric and the NSS perturbations. In the previous study using a NSS field [18], $\delta \tilde{T}_j^i = 0$ for $i \neq j$. That's why in [18] we got $\phi = \psi$. Here inequality between ψ and ϕ arises because of the extra $F^{\mu\nu}$ term in the energy-momentum tensor. We consider \tilde{F} to be a very small quantity and from here onwards we will write the equations up to the linear order in \tilde{F} .

ii) $\delta G_i^\eta = 8\pi G \delta T_i^\eta$: Using the last equation of (3.6) we get

$$\begin{aligned} \frac{2}{a^2} (\phi' + H\psi)_{,i} &= 8\pi G \left[\frac{1}{a^2} \varphi' \delta\varphi - \frac{1}{4a^2} (H\varphi^2) \psi - \frac{a^2}{8} \left(\frac{\psi\varphi^2}{a^4} \right)' - \frac{1}{8a^2} H \left(\delta\bar{\lambda}\lambda + \bar{\lambda}\delta\lambda \right) - \right. \\ &\quad \left. \frac{1}{4a^2} (\psi + \phi) H\varphi^2 + \frac{1}{8a^2} \phi' \varphi^2 \right]_{,i}, \end{aligned}$$

or,

$$(\phi' + H\psi) = 4\pi G a^2 \left(\frac{\varphi'^2}{a^2} \right) \frac{\delta\varphi}{\varphi'} - H\tilde{F}\phi - \left(\frac{\psi' - \phi'}{2} \right) \tilde{F} - \frac{\tilde{F}'}{2} \left(H \frac{\delta\varphi}{\varphi'} + \psi \right).$$

Substituting $\psi = (1 - 2\tilde{F})\phi$ in the right hand side of the above equation,

$$(\phi' + H\psi) \simeq 4\pi G a^2 (\varepsilon + p) \frac{\delta\varphi}{\varphi'} + \left[\left(H\tilde{F} \right)' - H^2 \tilde{F} - \frac{H\tilde{F}'}{2} \right] \frac{\delta\varphi}{\varphi'} - \left(H\tilde{F} + \frac{\tilde{F}'}{2} \right) \phi. \quad (3.7)$$

Again setting $\psi = (1 - 2\tilde{F})\phi$ in the left hand side and multiplying both sides by $\frac{a^2}{H}$ one may obtain,

$$\begin{aligned} \left(\frac{a^2}{H} \phi \right)' &\simeq \left[\frac{4\pi G a^4}{H^2} (\varepsilon + p) + \frac{a^2}{H} \left\{ \frac{\left(H\tilde{F} \right)'}{H} - H\tilde{F} - \frac{\tilde{F}'}{2} \right\} \right] \left(H \frac{\delta\varphi}{\varphi'} + \phi \right) + \\ &\quad \left(2H\tilde{F} - \frac{\left(H\tilde{F} \right)'}{H} \right) \frac{a^2 \phi}{H}. \end{aligned} \quad (3.8)$$

iii) $\delta G_\eta^\eta = 8\pi G \delta T_\eta^\eta = 8\pi G \delta \varepsilon$: Now the first equation in (3.6) implies

$$\Delta\phi - 3H(\phi' + H\psi) = 4\pi G a^2 \delta\varepsilon.$$

Using the expression of $(\phi' + H\psi)$ from equation (3.7) we get

$$\Delta\phi - 3H \left[4\pi Ga^2 (\varepsilon + p) \frac{\delta\varphi}{\varphi'} + \left\{ (H\tilde{F})' - H^2\tilde{F} - \frac{H\tilde{F}'}{2} \right\} \frac{\delta\varphi}{\varphi'} - \left(H\tilde{F} + \frac{\tilde{F}'}{2} \right) \phi \right] \simeq 4\pi Ga^2 \delta\varepsilon.$$

Similarly, using the expression of ψ from equation (3.7) in the expression of $\delta\varepsilon$ we get,

$$\begin{aligned} \delta\varepsilon \simeq & \varepsilon_{,X} \frac{2X}{H} \left[\left(H\frac{\delta\varphi}{\varphi'} + \phi \right)' - \left\{ (H\tilde{F})' - H^2\tilde{F} - \frac{H\tilde{F}'}{2} \right\} \frac{\delta\varphi}{\varphi'} + \left(H\tilde{F} + \frac{\tilde{F}'}{2} \right) \phi \right] - \\ & \varepsilon_{,X} \frac{3}{4a^2} H\varphi^2 \left(H\frac{\delta\varphi}{\varphi'} + \phi \right)' + \varepsilon_{,X} \frac{3}{4a^2} (\psi' - \phi') H\varphi^2 - \varepsilon_{,X} \frac{1}{4a^2} (\Delta\psi) \varphi^2 - \\ & 3H (\varepsilon + p) \frac{\delta\varphi}{\varphi'}. \end{aligned}$$

Finally using $\psi = (1 - 2\tilde{F})\phi$ in the above expression of $\delta\varepsilon$, up to linear order in \tilde{F} the Einstein's equation becomes,

$$\begin{aligned} (1 + \varepsilon_{,X}\tilde{F}) \Delta\phi \simeq & \left(4\pi Ga^2 \varepsilon_{,X} \frac{2X}{H} - \varepsilon_{,X} 3H\tilde{F} \right) \left(H\frac{\delta\varphi}{\varphi'} + \phi \right)' + \\ & \left(3H - 4\pi Ga^2 \varepsilon_{,X} \frac{2X}{H} \right) \left\{ \frac{(H\tilde{F})'}{H} - H\tilde{F} - \frac{\tilde{F}'}{2} \right\} \left(H\frac{\delta\varphi}{\varphi'} + \phi \right) - \\ & \left(3H - 4\pi Ga^2 \varepsilon_{,X} \frac{2X}{H} \right) \frac{(H\tilde{F})'}{H} \phi. \end{aligned} \quad (3.9)$$

In order to calculate the power spectrum for ϕ and $\delta\varphi$, we have to solve equations (3.8,3.9). These equations are highly coupled and one may need to decouple them. For simplicity we first write equations (3.8,3.9) in a different notations as below:

$$x' = A_1 y + B_1 x, \quad (3.10)$$

$$A_2 \Delta x = B_2 y' + C_2 y - D_2 x. \quad (3.11)$$

where,

$$\begin{aligned} x &= \left(\frac{a^2\phi}{H} \right), \\ y &= \left(H\frac{\delta\varphi}{\varphi'} + \phi \right), \\ A_1 &= \frac{4\pi Ga^4}{H^2} (\varepsilon + p) + \frac{a^2}{H} \left[\frac{(H\tilde{F})'}{H} - H\tilde{F} - \frac{\tilde{F}'}{2} \right], \\ B_1 &= \left(2H\tilde{F} - \frac{(H\tilde{F})'}{H} \right), \\ A_2 &= \left(1 + \varepsilon_{,X}\tilde{F} \right), \end{aligned}$$

$$\begin{aligned}
B_2 &= \frac{a^2}{H} \left(4\pi G a^2 \varepsilon_{,X} \frac{2X}{H} - \varepsilon_{,X} 3H\tilde{F} \right), \\
C_2 &= \frac{a^2}{H} \left(3H - 4\pi G a^2 \varepsilon_{,X} \frac{2X}{H} \right) \left[\frac{(\tilde{H}\tilde{F})'}{H} - H\tilde{F} - \frac{\tilde{F}'}{2} \right], \\
D_2 &= \left(3H - 4\pi G a^2 \varepsilon_{,X} \frac{2X}{H} \right) \frac{(\tilde{H}\tilde{F})'}{H}.
\end{aligned}$$

y can be eliminated from equation (3.11) by using equation (3.10) and the decoupled equation for x can be written as,

$$x'' - \frac{A_1 A_2}{B_2} \Delta x + \left[A_1 \left\{ \left(\frac{1}{A_1} \right)' - \frac{B_1}{A_1} \right\} + \frac{C_2}{B_2} \right] x' - \left[A_1 \left(\frac{B_1}{A_1} \right)' + C_2 \frac{B_1}{B_2} + D_2 \frac{A_1}{B_2} \right] x = 0. \quad (3.12)$$

Next, it is useful to substitute $x = u(\eta, \vec{x})f(\eta)$ in the above equation

$$\begin{aligned}
u'' - \frac{A_1 A_2}{B_2} \Delta u + \left[2 \frac{f'}{f} + \left\{ A_1 \left(\frac{1}{A_1} \right)' - B_1 + \frac{C_2}{B_2} \right\} \right] u' + \\
\left[\frac{f''}{f} + \left\{ A_1 \left(\frac{1}{A_1} \right)' - B_1 + \frac{C_2}{B_2} \right\} \frac{f'}{f} - \left\{ A_1 \left(\frac{B_1}{A_1} \right)' + C_2 \frac{B_1}{B_2} + D_2 \frac{A_1}{B_2} \right\} \right] u = 0.
\end{aligned}$$

By equating the coefficient of u' to zero one can gets

$$\begin{aligned}
f &= \exp \left[-\frac{1}{2} \int \left\{ A_1 \left(\frac{1}{A_1} \right)' - B_1 + \frac{C_2}{B_2} \right\} d\eta \right] \\
&= \sqrt{A_1} \exp \left[\frac{1}{2} \int \left(B_1 - \frac{C_2}{B_2} \right) d\eta \right]
\end{aligned}$$

Here f' and f'' can be written as

$$\begin{aligned}
\frac{f'}{f} &= -\frac{1}{2} \left[A_1 \left(\frac{1}{A_1} \right)' - B_1 + \frac{C_2}{B_2} \right] \\
\frac{f''}{f} &= \left[-\frac{1}{2} \left\{ A_1 \left(\frac{1}{A_1} \right)' - B_1 + \frac{C_2}{B_2} \right\} \right]^2 - \frac{1}{2} \left[A_1 \left(\frac{1}{A_1} \right)' - B_1 + \frac{C_2}{B_2} \right]'.
\end{aligned}$$

Finally the generalized Mukhanov-Sasaki equation can be written as

$$\begin{aligned}
u'' - \frac{A_1 A_2}{B_2} \Delta u + \left[-\frac{1}{4} \left\{ A_1 \left(\frac{1}{A_1} \right)' - B_1 + \frac{C_2}{B_2} \right\}^2 - \frac{1}{2} \left\{ A_1 \left(\frac{1}{A_1} \right)' - B_1 + \frac{C_2}{B_2} \right\}' - \right. \\
\left. \left\{ A_1 \left(\frac{B_1}{A_1} \right)' + C_2 \frac{B_1}{B_2} + D_2 \frac{A_1}{B_2} \right\} \right] u = 0,
\end{aligned}$$

which one may write in a more simplified form as

$$u'' + (1 + A) k^2 u - \left(\frac{\theta''}{\theta} + B \right) u = 0, \quad (3.13)$$

Here both A and B are functions of \tilde{F} and its derivatives. In the limit $\tilde{F} \rightarrow 0$ both $A, B \rightarrow 0$ and one recovers the standard Mukhanov-Sasaki equation for a canonical scalar-field [3].

The coefficient of the k^2 term in equation (3.13) can be regarded as the square of sound speed (c_s^2), which implies $c_s^2 = (1 + A) = \frac{A_1 A_2}{B_2}$. Using the expressions of A_1, A_2 and B_2 in terms of background quantities, we can write c_s^2 after some algebra :

$$c_s^2 \simeq 1 + \tilde{F} \left[1 - \frac{1}{3} \frac{\tilde{F}'}{H\tilde{F}} \frac{1}{(1 + \frac{p}{\varepsilon})_{\text{can}}} \right] \quad (3.14)$$

where, $(1 + \frac{p}{\varepsilon})_{\text{can}}$ can be obtained by setting \tilde{F} terms to zero in equations (2.4-2.5). On galactic scale $\frac{1}{(1 + \frac{p}{\varepsilon})_{\text{can}}} \sim 10^{-2}$ (for example one can see Ref. [3]). Again, in slow-roll inflation we can consider $\frac{\tilde{F}'}{H\tilde{F}} \ll 1$. Thus one can write $c_s^2 \simeq 1 + \tilde{F}$.

4 Calculation of power spectrum

From the solutions of equation (3.13) the power spectrum for the scalar-perturbations can be calculated. In what follows we closely follow the method of the power-spectrum calculations given in [3] for a canonical scalar-field.

i) Short wavelength(large k) region : For a short wavelength regime (or large k), we can neglect $\left(\frac{\theta''}{\theta} + B\right)$ term with respect to $(1 + A) k^2$ term in equation (3.13) and write

$$u'' + (1 + A) k^2 u = 0. \quad (4.1)$$

One may look for the solution of equation (4.1) in the form $u = c(\eta) \exp [ik \int \sqrt{1 + A} d\eta]$. Substituting this back into equation(4.1) we get a 2nd order equation for $c(\eta)$,

$$c'' + ikc' \left(1 + \frac{A}{2} \right) + ikc \frac{A'}{2} = 0, \quad (4.2)$$

where we have considered A to be a small quantity and write $\sqrt{1 + A} \simeq 1 + \frac{A}{2}$. Next, We look for an approximate solution of equation(4.2) by regarding A and A' to be small. Thus we consider $c \approx c_0 + \bar{c}$ with $|c_0| > |\bar{c}|$ and \bar{c} is of the same order of A and A' . Equations for c_0 and \bar{c} can be written as follows,

$$\begin{aligned} c_0'' + ikc_0' &= 0 \\ \bar{c}'' + ikc_0' \frac{A}{2} + ik\bar{c}' + ikc_0 \frac{A'}{2} &= 0. \end{aligned} \quad (4.3)$$

The solution for c_0 can be written as

$$c_0 = b_2 - \frac{b_1 e^{-ik\eta}}{ik},$$

where b_1 and b_2 are the constants of integration. Solution for \bar{c} can be obtained as

$$\bar{c} = e^{-ik\eta} \int (b_1 - ikb_2 e^{ik\eta}) \frac{A}{2} d\eta.$$

Finally we get,

$$c(\eta) = b_2 - \frac{b_1 e^{-ik\eta}}{ik} + e^{-ik\eta} \int (b_1 - ikb_2 e^{ik\eta}) \frac{A}{2} d\eta.$$

Since in the limit when $A = 0$ one should get the solution of the canonical scalar-field [3], we set $b_1 = 0$ and $b_2 = -\frac{i}{k^{\frac{3}{2}}}$. Thus one can write solution of equation (4.1) as

$$u = -\frac{i}{k^{\frac{3}{2}}} \left[1 - ike^{-ik\eta} \int e^{ik\eta} \frac{A}{2} d\eta \right] \exp \left[ik \int \sqrt{1+A} d\eta \right]. \quad (4.4)$$

Finally one can obtain

$$\begin{aligned} \phi = & -\frac{i}{k^{\frac{3}{2}}} \left[1 - ike^{-ik\eta} \int e^{ik\eta} \frac{A}{2} d\eta \right] \left\{ \frac{H}{a^2} \sqrt{A_1} \exp \left[\frac{1}{2} \int \left(B_1 - \frac{C_2}{B_2} \right) d\eta \right] \right\} \times \\ & \exp \left[ik \int \sqrt{1+A} d\eta \right]. \end{aligned} \quad (4.5)$$

From this the power spectrum for ϕ in case of large k (small wavelength) can be found to be

$$\begin{aligned} \delta_\phi^2 &= |\phi|^2 k^3 \\ &= \left\{ \frac{H^2}{a^4} A_1 \exp \left[\int \left(B_1 - \frac{C_2}{B_2} \right) d\eta \right] \right\} \left[1 - ike^{-ik\eta} \int e^{ik\eta} \frac{A}{2} d\eta \right]^2. \end{aligned} \quad (4.6)$$

The at large k (or small wavelength) scales the power spectrum of ϕ is not a scale-invariant. However it can become a scale invariant if A can be regarded as a constant.

ii) Large wavelength(Small k) region : In case of small k regime one can neglect $(1+A)k^2$ term with respect to $(\frac{\theta''}{\theta} + B)$ term. In this case we write the equation (3.13) can be written as

$$u'' - \left(\frac{\theta''}{\theta} + B \right) u = 0. \quad (4.7)$$

It is useful to look for the solution of u in the form $u = u_{can}g$ where, u_{can} is the solution when $B = 0$ i.e. no effect of non standard spinor is considered. This implies that in the $B \rightarrow 0$ limit $g = 1$. Now substituting for u into equation(4.7) we get the equation for g as

$$g'' + 2 \left(\frac{u'_{can}}{u_{can}} \right) g' - Bg = 0. \quad (4.8)$$

For the case when $B = B(\tilde{F})$ is a small quantity, an approximate solution of $g \approx (g_0 + \bar{g})$ with $|g_0| > |\bar{g}|$ can be obtained in a manner similar to that discussed in the previous section. From equation (4.8) we get

$$\begin{aligned} g_0'' + 2 \left(\frac{u'_{can}}{u_{can}} \right) g_0' &= 0 \\ \bar{g}'' + 2 \left(\frac{u'_{can}}{u_{can}} \right) \bar{g}' &= Bg_0. \end{aligned} \quad (4.9)$$

From the equation for g_0 we get

$$g_0 = c_1 + \int \left(\frac{c_2}{u_{can}^2} \right) d\eta,$$

where c_1 and c_2 are constants of integration. Plugging this solution of g_0 into the equation for \bar{g} and solving the inhomogeneous equation, we can write get \bar{g} as

$$\bar{g} = \int \frac{1}{u_{can}^2} \left[\int B u_{can}^2 d\eta \right] d\eta.$$

Therefore we get,

$$g = c_1 + \int \left(\frac{c_2}{u_{can}^2} \right) d\eta + \int \frac{1}{u_{can}^2} \left[\int B u_{can}^2 d\eta \right] d\eta.$$

Since $g = 1$ when $B = 0$, one can set $c_1 = 1$ and $c_2 = 0$. The approximate solution for u can be written as

$$u \simeq u_{can} \left(1 + \int \frac{1}{u_{can}^2} \left[\int B u_{can}^2 d\eta \right] d\eta \right). \quad (4.10)$$

Thus one can notice from the expression of u in the long wavelength(small k) regime that the resultant power spectrum is a scale invariant quantity. Therefore in a long wavelength regime we can write

$$\phi \simeq \frac{H}{a^2} \sqrt{A_1} \exp \left[\frac{1}{2} \int \left(B_1 - \frac{C_2}{B_2} \right) d\eta \right] u_{can} \left(1 + \int \frac{1}{u_{can}^2} \left[\int B u_{can}^2 d\eta \right] d\eta \right)$$

Finally we get power spectrum of ϕ as

$$\begin{aligned} \delta_\phi^2 &= |\phi|^2 k^3 \\ &= \delta_{\phi(can)}^2 \left[1 - \frac{H \tilde{F}'}{8\pi G a^2 (\varepsilon + p)_{can}} \right] \exp \left[\int \left(B_1 - \frac{C_2}{B_2} \right) d\eta \right] \times \\ &\quad \left(1 + \int \frac{1}{u_{can}^2} \left[\int B u_{can}^2 d\eta \right] d\eta \right)^2 \end{aligned} \quad (4.11)$$

Now as $\int \frac{1}{u_{can}^2} \left[\int B u_{can}^2 d\eta \right] d\eta$ are k independent, we get that power spectrum of ϕ for large wavelength(small k) is scale independent. Taking logarithm on both side we get

$$\begin{aligned} \ln \delta_\phi^2 &= \ln \delta_{\phi(can)}^2 + \ln \left[1 - \frac{H \tilde{F}'}{8\pi G a^2 (\varepsilon + p)_{can}} \right] + \left[\int \left(B_1 - \frac{C_2}{B_2} \right) d\eta \right] + \\ &\quad 2 \ln \left(1 + \int \frac{1}{u_{can}^2} \left[\int B u_{can}^2 d\eta \right] d\eta \right). \end{aligned}$$

Spectral index for scalar perturbation can be written as

$$n_s - 1 = \frac{d \ln (\delta_\phi^2)}{d \ln k}.$$

At the time of horizon crossing ($c_s k = aH$), derivative with respect to $\ln k$ can be approximated as $d \ln k = \frac{1}{H} d\eta$ (here we have considered that variation of sound velocity and Hubble parameter are very small, therefore can be neglected). Therefore in the expression for the spectral index all the logarithmic derivatives can be replaced with time derivatives and finally we get

$$n_s - 1 = \frac{1}{H} \left(\ln \delta_{\phi(\text{can})}^2 \right)' + \frac{1}{H} \left(\ln \left[1 - \frac{H\tilde{F}'}{8\pi G a^2 (\varepsilon + p)_{\text{can}}} \right] \right)' + \frac{1}{H} \left(B_1 - \frac{C_2}{B_2} \right) + \frac{2}{H} \left[\ln \left(1 + \int \frac{1}{u_{\text{can}}^2} \left[\int B u_{\text{can}}^2 d\eta \right] d\eta \right) \right]' . \quad (4.12)$$

In the case of a slow roll if A is a measurable quantity then $\frac{A'}{HA}$ is very small and it can be neglected. So we argue that in the above expression we can neglect the second and last term. In the case of a canonical scalar-field we can write the first term in equation (4.12) following Ref.[3] as

$$\frac{1}{H} \left(\ln \delta_{\phi(\text{can})}^2 \right)' \simeq -3 \left(1 + \frac{p}{\varepsilon} \right)_{\text{can}} . \quad (4.13)$$

But in the case of NSS the correction term $\frac{1}{H} (B_1 - \frac{C_2}{B_2})$ can be approximated as

$$\frac{1}{H} \left(B_1 - \frac{C_2}{B_2} \right) \simeq \frac{3H^2}{4\pi G \varphi'^2} \tilde{F} .$$

Using the Friedmann's equation and keeping the terms up to linear order in \tilde{F} we write

$$\frac{1}{H} \left(B_1 - \frac{C_2}{B_2} \right) \simeq 2 \frac{1}{\left(1 + \frac{p}{\varepsilon} \right)_{\text{can}}} \tilde{F} . \quad (4.14)$$

Finally using (4.13) and (4.14) we get spectral index for scalar perturbation as

$$n_s = 1 - 3 \left(1 + \frac{p}{\varepsilon} \right)_{\text{can}} + 2 \frac{1}{\left(1 + \frac{p}{\varepsilon} \right)_{\text{can}}} \tilde{F} . \quad (4.15)$$

On galactic scale the canonical terms $\left(1 + \frac{p}{\varepsilon} \right)_{\text{can}}$ can be estimated as 10^{-2} and ε_{can} can be estimated as 10^{-12} of the Planckian density[3]. Then equation (4.15) can be written as

$$n_s - 1 = -0.03 + 200 \tilde{F} . \quad (4.16)$$

WMAP 7 years data suggests $n_s = 0.968 \pm 0.012$ with 68 % CL [28]. Therefore from equation (4.16) we can understand that, to get n_s closer to the observed value, \tilde{F} has to be smaller than 10^{-4} . \tilde{F} is the only new feature which NSS driven inflation brings over the inflationary scenario driven by canonical scalar field. Although \tilde{F} is not a part of potential in the theory, its value may be estimated from V . As the potential $V(\varphi)$ is the dominant term in ε_{can} , we can write $\varepsilon_{\text{can}}/\varepsilon_{\text{PL}} \sim \frac{V}{\varepsilon_{\text{PL}}} \sim \frac{V}{M_{\text{PL}}^4} \sim 10^{-12}$. Now from different models of potentials we can estimate \tilde{F} . For example, if we consider φ^4 kind of potential then $\varepsilon_{\text{can}}/\varepsilon_{\text{PL}}$ becomes \tilde{F}^2 and from the value of ε_{can} we can estimate $\tilde{F} \sim 10^{-6}$ which is consistent with the NSS model. Upcoming experiments like PLANCK[29] can further

constrain \tilde{F} by measuring n_s more accurately.

In the case of a canonical scalar-field inflation it is well known that at a large scale the power-spectrum of tensor perturbation is [3] $\delta_{h(\text{can})}^2 \simeq \frac{8}{\pi} H^2$. But for the present case the power-spectrum for the tensor perturbation is modified to

$$\delta_h^2 \simeq \frac{8}{\pi} H^2 \times f(\tilde{F}). \quad (4.17)$$

Thus when $\tilde{F} \rightarrow 0$, $f(\tilde{F}) \rightarrow 1$ and we get the power-spectrum of the tensor perturbations for a canonical scalar-field. Since \tilde{F} is a small quantity, the tensor to scalar ratio of the power spectrum for a NSS still be very small.

In conclusion, we have studied the cosmological perturbations generated by the inflation driven by a Lorentz invariant NSS model. We find that the the usual condition for the gravitational potentials ϕ and ψ for scalar-perturbations i.e. $\delta T_j^i = 0$ giving $\psi = \phi$ is modified to $\psi = (1 - 2\tilde{F})\phi$. We have also shown that the perturbations are nearly scale invariant and the hedgehog ansatz is not required. More importantly we have calculated the power-spectrum and spectral index for the metric perturbation. The model predicts the running spectral index which allows for a wide range of \tilde{F} . For the case $\tilde{F} = 0$ one gets back the expressions for the power spectrum and spectral index for a canonical scalar-field. Further our analysis shows that the calculated value of the spectral index n_s can match to the value obtained from the WMAP data if there is an upper bound on the parameter $\tilde{F} < 10^{-4}$. Our analysis shows that the sound speed of the perturbation is not a constant but dependent on time. However, the expression of $c_s^2 \simeq 1 + \tilde{F}$ and the upper bound on \tilde{F} imply that $c_s^2 \sim 1$. Finally the tensor to scalar ratio of the power spectrum remains much smaller as in the case of a scalar-field inflation due the upper bound on \tilde{F}

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